Looking for a new version of Gordon's identities And Differential ideals

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Abstract: A *partition* (of length ℓ) of a positive integer n is a sequence $\Lambda : (\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_\ell)$ of positive integers λ_i, for $1 \leq i \leq \ell$, such that

$$\lambda_1 + \cdots + \lambda_\ell = n.$$

The integers λ_i are called the *parts of the partition* Λ. The partitions identities, which stipulate that the number of the partitions of an integer n satisfying a certain condition A is equal to the number of the partitions of n satisfying another condition B, play an important role in many areas like number theory, combinatorics, Lie theory, particle physics and statistical mechanics. One family of important partitions identities is called *Gordon's identities*:

Theorem. (*Gordon's identities*). Given integers $r \geq 2$ and $1 \leq i \leq r$, let $B_{r,i}(n)$ denote the number of partitions of n of the form $(b_1, \ldots, b_s)$, where $b_j - b_{j+r-1} \geq 2$ and at most $i-1$ of the b_j are equal to 1. Let $A_{r,i}(n)$ denote the number of partitions of n into parts $\not\equiv 0, \pm i \pmod{2r+1}$. Then $A_{r,i}(n) = B_{r,i}(n)$ for all integer n.

Using differential ideals, we can conjecture a family of partition identities related to Gordon's identities. We prove this conjecture for two identities among this family.